

14.25. Model: Assume the pendulum to have small-angle oscillations. In this case, the pendulum undergoes simple harmonic motion.

Solve: Using the formula $g = GM/R^2$, the periods of the pendulums on the moon and on the earth are

$$T_{\text{earth}} = 2\pi \sqrt{\frac{L}{g}} = 2\pi \sqrt{\frac{L_{\text{earth}} R_{\text{earth}}^2}{GM_{\text{earth}}}} \quad \text{and} \quad T_{\text{moon}} = 2\pi \sqrt{\frac{L_{\text{moon}} R_{\text{moon}}^2}{GM_{\text{moon}}}}$$

Because $T_{\text{earth}} = T_{\text{moon}}$,

$$\begin{aligned} 2\pi \sqrt{\frac{L_{\text{earth}} R_{\text{earth}}^2}{GM_{\text{earth}}}} &= 2\pi \sqrt{\frac{L_{\text{moon}} R_{\text{moon}}^2}{GM_{\text{moon}}}} \Rightarrow L_{\text{moon}} = \left(\frac{M_{\text{moon}}}{M_{\text{earth}}} \right) \left(\frac{R_{\text{earth}}}{R_{\text{moon}}} \right)^2 L_{\text{earth}} \\ &= \left(\frac{7.36 \times 10^{22} \text{ kg}}{5.98 \times 10^{24} \text{ kg}} \right) \left(\frac{6.37 \times 10^6 \text{ m}}{1.74 \times 10^6 \text{ m}} \right)^2 (2.0 \text{ m}) = 33 \text{ cm} \end{aligned}$$